SEMESTRAL EXAMINATION
B. MATH III YEAR, I SEMESTER 2013-2014
PROBABILITY III

The 6 questions carry a total of 110 marks. The maximum you can score is 100. Time limit is 3hrs.

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1. Show that any two communicating states have the same period. [20]

2. Let $P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$. If $\{X_n\}$ is a Markov chain with transition matrix P show that all states are positive recurrent and find the mean return time for each state. [25]

3. If P is an $N \times N$ doubly stochastic matrix prove that the each state is positive recurrent and the mean return time to each state is N. [10]

4. Let $\{X_t\}_{t \geq 0}$ be a birth and death process with state space $\{0, 1\}$. Find $P\{X_t = 0 | X_0 = 1\}$ in terms of the birth and death rates. [25]

5. Let Q be obtained from a primitive transition matrix P by replacing one row by the zero vector. If λ is an eigen value of Q show that $|\lambda| < 1$. [10]

6. If P is an $N \times N$ transition matrix such that $\displaystyle\sum_{j=1}^{N} j p_{ij} = i \ (1 \leq i \leq N)$ prove that 1 and N are absorbing . Determine the probability of absorption from state 2 to state 1. [20]